Mark schemes

1 (a) (i) speed at P, v (= $\sqrt{2gh}$) = $\sqrt{2 \times 9.81 \times 25}$ ✓

= 22(.1) (m s^{-1}) ✓

2

(ii) use of $F = k\Delta L$ gives $d \left(= \frac{F}{K}\right) = \frac{58 \times 9.81}{54}$ ✓

= 11 (10.5) (m) ✓

2

(b) (i) period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{58}{54}}$ ✓ (= 6.51 s)

time for one half oscillation = 3.3 (3.26) (s) ✓

2

(ii) frequency $f \left(= \frac{1}{T}\right) = \frac{1}{6.51}$ ✓ (= 0.154 (Hz))

use of $v = \pm 2\pi f\sqrt{A^2 - x^2}$ when x = 10.5 m and v = 22.1 m s^{-1} gives 22.1^2

$= 4\pi^2 \times > 0.154^2 (A^2 - 10.5^2)$ ✓

from which A = 25.1 (m) ✓

[alternatively, using energy approach gives $\frac{1}{2} m v_P^2 + mg\Delta L = \frac{1}{2} k(\Delta L)^2$ ✓

$\therefore (29 \times 22.1^2) + (58 \times 9.81 \times \Delta L) = 27 (\Delta L)^2$

solution of this quadratic equation gives ΔL = 35.7 (m) ✓

from which A = 25.2 (m) ✓]

3

(c) bungee cord becomes slack ✓

student's motion is under gravity (until she returns to **P**) ✓

has constant downwards acceleration **or** acceleration is not ∝ displacement ✓

2

(d) (i) when student is at **R** or at bottom of oscillation ✓

1

(ii) at uppermost point **or** where it is attached to the railing ✓

because stress = F/A and force at this point includes weight of whole cord ✓

[accept alternative answers referring to mid-point of cord because cord will show thinning there as it stretches **or** near knots at top or bottom of cord where A will be smaller with a reference to stress = F/A]

2

[14]

2 (a) acceleration is proportional to displacement **(1)**
acceleration is in opposite direction to displacement, or towards a fixed point, or towards the centre of oscillation **(1)**

2

(b) (i) $f = \frac{25}{23}$ = 1.1 Hz (or s^{-1}) **(1)** (1.09 Hz)

(ii) (use of $a = (2\pi f)^2 A$ gives)
$a = (2\pi \times 1.09)^2 \times 76 \times 10^{-3}$ **(1)**
= 3.6 m s^{-2} **(1)** (3.56 m s^{-2})
(use of f = 1.1 Hz gives a = 3.63 m s^{-2})
(allow C.E. for incorrect value of f from (i))

(iii) (use of $x = A\cos(2\pi ft)$ gives)
$x = 76 \times 10^{-3} \cos(2\pi \times 1.09 \times 0.60)$ **(1)**
= (–)4.3(1) × 10^{-2}m **(1)** (43 mm)
(use of f = 1.1 Hz gives
x = (–)4.0(7) × 10^{-2} m (41 mm))
direction: above equilibrium position or upwards **(1)**

6

(c) (i) graph to show:
correct shape, i.e. cos curve **(1)**
correct phase i.e. –(cos) **(1)**

(ii) graph to show:
two cycles per oscillation **(1)**
correct shape (even if phase is wrong) **(1)**
correct starting point (i.e. full amplitude) **(1)**

max 4

[12]

3 (a) (i) acceleration (not a) and displacement (not x) are in opposite directions OR restoring force/acceleration always acts toward rest position

B1

1

(ii) (+) sine curve consistent with a graph

B1

1

(b) (i) statement that $E_K = E_P$

B1

statement of max values considered

B1

$E_P = ½\, k(\Delta l)^2$ or $E_{Pmax} = ½\, kA^2$

B1

correctly substituted values

B1

$E_K = 3.7 \times 10^{-2}$ J

B1

OR
$f = 1/T$ or $T = 3.97$ s or period equation

B1

leading to $f = 0.252$ Hz

B1

$\omega_{max} = 1.58$ rad s^{-1} or $v_{max} = 0.055 ms^{-1}$ (seen or used)

B1

substituted values into $E_K = ½mA^{2\omega 2}$ or $E_K = ½mv^2$

B1

$E_K = 3.7 \times 10^{-2}$ J

B1

5

(ii) any attenuation from $t = 0$ seen

M1

10 mJ or $E_0/4$ at either 4s or third hump

M1

consistent period values minima at 1 and 3s
maxima at 0 and 4s

A1

3

[10]

4 (a) acceleration/force is directed toward
a (fixed) point/the centre/the equilibrium position
or
$a = -kx$ + '–' means that a is opposite direction to x

B1

acceleration/force is proportional to the distance from the point/displacement
or
$a = -kx$ where a = acceleration; x = displacement and
k is constant

B1

2

(b) (i) $3.2 = 2\pi\sqrt{l/9.8}$ (condone use of $g = 10$ m s^{-2} for C mark)
(use of $a = -\omega^2 x$ is a PE so no marks)

C1

2.5(4) m

A1

2

(ii) Correct value at 0.5 m and correct curvature

M1

Energy at 1 m = 160 J

A1

2

[6]

5 (a) Max to zero to max with zero at 0 displacement and correct amplitude correct shape drawn with reasonable attempt to keep total energy constant, crossing at 1×10^{-2} J

A1

(2)

(b) (i) 0.044 m

B1 **(1)**

(ii) $x = 0.044 \cos 2\pi 3.5t$ ($0.044 \cos 22t$) or $x = 0.044 \sin 2\pi 3.5t$ etc
ecf for A

B1 **(1)**

(iii) $\alpha_{max} = (2\pi 3.5)^2\ 0.044$

C1

21 (21.3) m s^{-2} ecf for A and incorrect $2\pi f$ from (ii)
(0.042 gives 20.3; 0.04 gives 19.4)

A1 **(2)**

[6]

6 (a) (i) Maximum **displacement** (of carriage/pendulum from rest position)

B1 **1**

(ii) 6.0 (m)

B1 **1**

(iii) Clear evidence of what constitutes period

C1

4.8–4.9 (s)

A1 **2**

(b) (i) Use of $v = 2\pi fA$

C1

7.07 (ms^{-1})

A1 **2**

(ii) Use of $a = 4\pi^2 f^2 A$

C1

11.1 (ms^{-2}) ecf

A1 **2**

(iii) Substitution into or rearrangement of $T = 2\pi\sqrt{l/g}$

C1

3.98 (m)

A1

2

(c) Applied frequency = natural frequency

B1

Mention or clear description of resonance

B1

2

(d) Resistive/frictional/damping/air resistance forces

C1

due to friction in named place (eg in bearings)/air resistance acting on named part (allow ride/gondola here)

A1

low friction/large mass or inertia /streamline/smooth surface etc.

B1

3

[15]

7 D

[1]

8 **D**

[1]

9 D

[1]

10 D

[1]

Examiner reports

1 This question, based on bungee jumping, tested simple harmonic motion in an unfamiliar context and at the same time to provide a synoptic test of some AS content. Examiners were pleased to see that a high proportion of the students were able to cope competently with this unfamiliar situation.

Application of energy conservation, or of the equations for uniformly accelerated motion under gravity, led to a high proportion of correct answers in part (a) (i). The equation representing Hooke's law was well known in part (a) (ii) but a few students showed confusion between mass and weight.

Part (b) (i), which required the time for half of an oscillation, only caused problems for the small number of students who misinterpreted the wording and determined the time for one-and-a-half oscillations. Part (b) (ii) was much more challenging and turned out to be a question that many students returned to answer on a supplementary sheet. The most direct solution came by applying the equation $v = \pm 2\pi f \sqrt{A^2 - x^2}$, with careful choice of the earlier values obtained for v and x, and of the derived value for f. Most students seemed to think a quick solution could be arrived at by applying $v_{max} = 2\pi fA$, but this is incorrect. It is possible to reach a correct solution from energy considerations; this needs particular care over the balance of gravitational pe lost, ke gained and elastic pe gained at some consistent point in the motion. Nevertheless, a few correct solutions using this approach were seen.

In part (c) most students realised that the bungee cord would cease to exert a force on the bungee jumper once she was higher than point P. Few went on to mention that her motion was then purely under gravity or that her acceleration became constant, although references to the fact that acceleration would no longer be proportional to displacement were quite common.

Almost all students gave the correct answer – point R – in part (d) (i). The responses in part (d) (ii) revealed a widespread misunderstanding of the significance of centre of mass, with statements such as 'the stress is a maximum at the centre of the cord because that is where the weight acts' seen. Acceptable answers included at the point where the cord is attached to the railing (where the greatest weight is supported) and (because of possible thinning) half way along the cord. It was expected that students would show that they understood what is meant by stress when formulating their reason, whichever point in the cord they gave.

2 The conditions expected in answers to part (a) were those embodied in the definition of shm: acceleration is proportional to displacement, but acts in the opposite direction (or towards a fixed point / towards equilibrium position). Other features of the acceleration, such as the fact that a is a maximum when $v = 0$, were not given any credit.

In part (b) frequency was often confused with period; of itself this was only penalised once, leaving five marks available. Part (b) (iii) caused problems for many candidates, mainly because they did not realise that $2\pi ft$ is an angle measured in radians rather than degrees. Several candidates confused acceleration a and amplitude A, leading to the incorrect substitution $x = 3.56 \cos (2\pi \times 1.09 \times 0.60)$. Another prevalent wrong answer was 49 or 50 mm, apparently arrived at by calculating (0.60/0.92) × 76 mm, i.e. $(t/T) \times A$. The direction of displacement when t = 0.60s could be arrived at heuristically, without resort to the result of the previous calculation; the direction mark was therefore regarded as independent.

Many very good answers were seen for the graphs in part (c). Common errors were a cos graph (rather than – cos) in (i), and the wrong shape of E_p curve – even when it had been appreciated that there are two energy cycles per oscillation – in (ii).

3 (a) (i) The clear majority of the candidates were able to explain that the minus sign showed either that the acceleration was directed towards the equilibrium position or else that it indicated that the acceleration and the displacement were in opposite directions. Weaker candidates suggested that it meant that there was always deceleration or that the acceleration was in the opposite direction to the 'motion'.

(iii) Most candidates recognised that the velocity would be a positive sine curve, although other sinusoidal curves were all relatively common.

(b) (i) The two routes to solution of this part were equally common. Those candidates opting for the 'elastic potential energy' version often failed to explain that in equating the elastic potential energy to the kinetic energy, they were using the maximum values. Those candidates calculating the kinetic energy through the frequency or angular frequency, often missed out the intermediate steps that a 'show that' equation requires. This second method gave the period that was needed to answer part (b)(ii).

(ii) The majority of candidates found this part to be difficult, with very few scoring all three marks. Although virtually all candidates attempting this part were credited for showing attenuation of the signal, most did not recognise that after one period the energy would have fallen to a quarter of the maximum value when the amplitude fell to half its maximum value. Very few candidates drew an accurate curve showing this.

4 (a) The majority appreciated what was required but many failed to express one or both conditions clearly. Confused candidates referred to acceleration being proportional to and in the opposite direction to velocity or amplitude. A significant proportion stated only that for SHM the period is independent of amplitude which gained no marks.

(b) (i) This was generally well done. Some inappropriately used the SHM equation $g = -(2\pi/3.2)^2x$ in which x was equal to the length of the string. This gave the correct numerical value but the use of wrong physics resulted in no marks.

(ii) There were few correct answers to this part. Many who were on the right lines failed to get the value at 1.00 m correct. There were many who simply drew a straight line through the origin. Careless reading led many to decide that this question was about the total energy remaining constant during one cycle of SHM. Others drew graphs showing the variation of KE or PE (or both) with displacement.

5 (a) The majority of the candidates gained full credit for this question. The most common cause of a lost mark was failure to show clearly that the total energy was constant. This was judged by showing that KE and PE were equal at 1×10^{-2} J.

(b) (i) Misreading the amplitude as 0.042 m or giving the maximum energy, 2×10^{-2} J were common errors.

(ii) There were many correct responses. There was a significant proportion, however, who did not understand the question and gave the formula for the period of the pendulum.

(iii) Allowing for errors carried forward this part was well done by most candidates. Giving the unit as m s^{-1} was a commonly seen error.

6 Many students failed to define amplitude precisely in part (a)(i). Many described displacement not amplitude and the word ‘maximum’ was frequently omitted. Most students gained the mark for part (a)(ii) but 12 was a relatively common incorrect response. Most students used multiple oscillations to allow the period to be calculated reasonably precisely in (a)(iii) – a minority tried to use the simple pendulum formula using the amplitude as the length.

All parts of (b) were almost invariably done well. A minority of students failed to square the 2π factor in part (b)(iii).

Most students recognised that part (c) was a resonance situation and either stated this or else described the effect in terms of energy transfer.

The effects of friction at the moving parts or air resistance acting on the gondola were usually mentioned in part (d); most students went on to explain that lubrication or a streamline shape reduced these factors and increased the time that the gondola would naturally come to rest in.

8 Two-thirds of the students selected the correct answer in this question, concerning the characteristics of simple harmonic motion. Distractor B (acceleration out of phase with velocity by 180°) and distractor C (velocity always in the same direction as displacement) both attracted significant numbers of responses.

9 This question on simple harmonic motion, readily gave the correct answer to students who could apply $a = -(2\pi f)^2 x$: clearly therefore a_{max} is proportional to x_{max}, the amplitude. 78% of the responses were correct. Incorrect answers were fairly evenly distributed amongst the other three distracters.

10 This question required the selection of a correct acceleration-displacement graph for a body moving with shm. 70% chose the correct one – a straight line of negative gradient – but 19% opted for the parabolic shape shown by distractor A. The facility of this question improved by 25% since it was used in a 2011 examination.