Mark schemes

1 (a) (use of $v = 2\pi f\sqrt{a^2 - x^2}$)

v_{max}=2π × 2.0 × 2.5 × 10^{-2}

v_{max}= 0.314 m s^{-1} ✓

(use of E_k = ½mv^2)

54 × 10^{-3} = ½m × $(0.314)^2$

m = 1.1 (kg) ✓

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

2.0 × 2π=√(k/1.1) ✓

($k = (4\pi)^2 \times 1.1$)

k = 173 (172.8) ✓(N m^{-1})

Can

OR

5.4 × 10^{-3} = ½ *k* $(2.5 \times 10^{-2})^2$ ✓

k = 173 (172.8) N m^{-1} ✓

If either of these methods used can then find mass from frequency formula or from kinetic energy

OR

54 × 10^{-3} = ½ *F* × 2.5 × 10^{-2}

F = 4.32

4.32 = *k* × 2.5 × 10^{-2}

k = 173 (N m^{-1})

Accept 170 and 172.8 to 174

1
1
1
1

(b) (use of $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$))

same mass so $f \propto \sqrt{k}$

thus frequency = $2.0 \times \sqrt{3}$

frequency = 3.5 (3.46) (Hz) ✓

Allow CE from (a) for k or m

1
1

(c) Two from:

(resonance) peak / maximum amplitude is at a higher frequency ✓
due to higher spring constant ✓

(resonant) peak would be broader ✓
due to damping ✓

amplitude would be lower (at all frequencies) ✓
due to energy losses from the system ✓

First mark in each case for effect
Second mark for reason
2 marks max for effects
2 marks max for reason
Cannot award from sketch graph unless explained
First mark in each pair stand alone
Second mark conditional on first in each pair

1
1
1
1

[10]

2 (a) SHM is when

The acceleration is proportional to the displacement ✓

the acceleration is in opposite direction to displacement✓

2

(b) f = 1/T =1/0.05=20 Hz ✓

($(v_{max} = 2\pi fA)$

$A = \frac{0.044}{2\pi \times 20}$ ✓ (=3.5 × 10^{-4} m)

2

(c) Cosine shape drawn, maximum at *t*= 0, amplitude 3.5 × 10^{-4} m ✓

1

(d) (any of the following when the velocity is zero) 0.00s, 0.025s,0.050s or 0.075s ✓

1

(e) when the vibrating surface accelerates <u>down</u> with an acceleration less than the acceleration of free fall the sand stays in contact. ✓

above a particular frequency, the acceleration is greater than g ✓

there is no contact force on the sand ***OR***

sand no longer in contact when downwards acceleration of plate is greater than acceleration of sand due to gravity ✓

3

(f) (when the surface acceleration is the same as free fall)

$g = r\,\varpi^2 = A\,(2\,\pi f)^2$ ✓

$f = \sqrt{(g / A 4\pi^2)} = (9.81/(3.5 \times 10^{-4} \times 4\,\pi^2))^{1/2} = 26.6(7)$ Hz ✓

2

[11]

3 (a) (i) elastic potential energy **and** gravitational potential energy ✓

For elastic pe allow "pe due to tension", or "strain energy" etc.

1

(ii) elastic pe → kinetic energy → gravitational pe → kinetic energy → elastic pe ✓✓

[**or** pe→ke→pe→ke→pe is ✓ only]

[**or** elastic pe → kinetic energy → gravitational pe is ✓ only]

If kinetic energy is not mentioned, no marks.

Types of potential energy must be identified for full credit.

2

(b) (i) period = 0.80 s ✓

during one oscillation there are two energy transfer cycles

(or elastic pe→ke→gravitational pe→ke→elastic pe in 1 cycle)

or there are two potential energy maxima per complete oscillation ✓

Mark sequentially.

2

(ii) sinusoidal curve of period 0.80 s ✓

– cosine curve starting at t = 0 continuing to t = 1.2s ✓

For 1st mark allow ECF from T value given in (i).

2

(c) (i) use of $T = 2\pi\sqrt{\frac{m}{k}}$ gives $0.80 = 2\pi\sqrt{\frac{0.35}{k}}$ ✓

$\therefore k\left(= \frac{4\pi^2 \times 0.35}{0.80^2}\right) = 22\ (21.6)$ ✓ N m^{-1} ✓

Unit mark is independent: insist on N m^{-1}.
Allow ECF from wrong T value from (i): use of 0.40s gives 86.4 (N m^{-1}).

3

(ii) maximum ke = (½ mv_{max}^2) = 2.0 × 10^{-2} gives

$v_{max}^2 = \frac{2.0 \times 10^{-2}}{0.5 \times 0.35}$ ✓ (= 0.114 m^2s^{-2}) and v_{max} = 0.338 (m s^{-1}) ✓

$v_{max} = 2\pi f A$ gives $A = \frac{0.338}{2\pi \times 1.25}$ ✓

and A = 4.3(0) × 10^{-2} m ✓ i.e. about 40 mm

[**or** maximum ke = (½ mv_{max}^2) = ½ $m\,(2\pi f A)^2$ ✓
½ × 0.35 × $4\pi^2$ × 1.25^2 × A^2 = 2.0 × 10^{-2} ✓

$\therefore A^2 = \frac{2 \times 2.0 \times 10^{-2}}{4\pi^2 \times 0.35 \times 1.25^2}$ ✓ (= 1.85 × 10^{-3})

and A = 4.3(0) × 10^{-2} m ✓ i.e. about 40 mm]
[**or** maximum ke = maximum pe = 2.0 × 10^{-2} (J)
maximum pe = ½ $k\,A^2$ ✓
∴ 2.0 × 10^{-2} = ½ × 21.6 × A^2 ✓

from which $A^2 = \frac{2 \times 2.0 \times 10^{-2}}{21.6}$ ✓ (= 1.85 × 10^{-3})

and A = 4.3(0) × 10^{-2} m ✓ i.e. about 40 mm]

First two schemes include recognition that f = 1 / T i.e. f = 1 / 0.80 = 1.25 (Hz).

Allow ECF from wrong T value from (i) – 0.40s gives A = 2.15 × 10^{-2}m but mark to max 3.

Allow ECF from wrong k value from (i) –86.4Nm^{-1} gives A = 2.15 × 10^{-2}m but mark to max 3.

4

[14]

4 (a) acceleration is proportional to displacement (from equilibrium) ✓

Acceleration proportional to negative displacement is 1st mark only.

acceleration is in opposite direction to displacement

or towards a fixed point / equilibrium

Don't accept "restoring force" for accln.

position ✓

2

(b) (i) $f\left(=\frac{1}{2\pi}\sqrt{\frac{g}{l}}\right)=\frac{1}{2\pi}\sqrt{\frac{9.81}{0.984}}$ ✓ $=$ 0.503 (0.5025) (Hz) ✓

3SF is an independent mark.

[**or** $T\left(=2\pi\sqrt{\frac{l}{g}}\right)=2\pi\sqrt{\frac{0.984}{9.81}}$ ✓ (= 1.9(90) (s))

When g = 9.81 is used, allow either 0.502 or 0.503 for 2nd and 3rd marks.

$f\left(=\frac{1}{T}\right)=\frac{1}{1.990}$ = 0.503 (0.5025) (Hz) ✓]

***Use of g = 9.8** gives 0.502 Hz: award only 1 of first 2 marks if quoted as 0.502, 0.503 0.50 or 0.5 Hz.*

answer to **3SF** ✓

3

(ii) $a\left(=-(2\pi f)^2 x\right)=(-)(2\pi\times 0.5025)^2\times 42\times 10^{-3}$ ✓

*Allow ECF from **any** incorrect f from (b)(i).*

= 0.42 (0.419) (m s^{-2}) ✓

2

(c) recognition of 20 oscillations of (shorter) pendulum

and / or 19 oscillations of (longer) pendulum ✓

Explanation: difference of 1 oscillation or phase change of 2π

or Δt = 0.1 so n = 2 / 0.1 =20, **or** other acceptable point ✓

time to next in phase condition = 38 (s) ✓

Allow "back in phase (for the first time)" as a valid explanation.

[**or** (T = 1.90 s so) (n + 1) × 1.90 = n × 2.00 ✓

gives n = 19 (oscillations of longer pendulum) ✓

minimum time between in phase condition = 19 × 2.00 = 38 (s) ✓]

3

5 (a) (i) Tension minimum at extremities or maximum at middle / bottom

Tension depends on (component of) weight and required centripetal force / velocity

Increases as acrobat moves downwards

Tension at bottom = $mg + mv^2/r$ or Tension = weight + centripetal force

Tension at extremity = $mg/\cos\theta$ (θ is angle between rope and vertical)

Max 3

3

(ii) Use of $T = 2\pi\sqrt{(l/g)}$

3.6 (3.59) (m)

Allow for change of subject for use

2

(b) (i) Frequency of swing = 0.26 Hz

Use of $v = 2\pi fA$

3.0 or 2.97 (m s^{-1})

alternative method

Change in pe = gain in ke

Calculating Δh by geometry from swing = 0.48 m

3.1 or 3.06 (m s^{-1})

3

(ii) Use of $s = \frac{1}{2} at^2$

time to reach safety net = 1.11 s

s = their answer to **(b)(i)** × their time to reach the net = answer

(answer is 3.3 m if all correct)

Allow for change of subject for use

3

(c) (i) Attempt at valid test:

Fractional change in amplitude for same time interval

or use of 'half life' method

or use of exponential formula ($A = A_o e^{-kt}$) to show that k is constant

Correct calculation for one pair of amplitudes

Correct for second pair and conclusion

for half life method must see curve through peaks or other indication to find values between peaks

3

(ii) Period shorter

Centre of mass of trapeze artist was lower than the bar

Effective length of the pendulum is lower

Bar likely to be low mass now have a pendulum with distributed mass / no longer a simple pendulum / centre of mass is half way along suspending rope

Calculates new effective length of the pendulum (2 m)

Max 2

2

[16]

6 (a) (i) correct period read from graph or use of $f=1/T$ 0.84±0.01 **C1**

2.4 Hz gets C1

correct frequency 1.2 (1.18 − 1.25 to 3 sf) **A1**

(ii) correct shape (inverse) **B1**

Crossover PE = KE **B1**

(b) (i) Use of $T = 2\pi\sqrt{\frac{l}{g}}$ **C1**

48.7 (49) m **A1**

(ii) v = 120 000 / 3600 = 33(.3) m s^{-1} **B1**

Use of $F = m\, v^2/r$ (allow v in km h^{-1}) **B1**

Total tension = 6337 + (280 × 9.81) = 9.083 × 10^3 N
Allow their central force **B1**

Divide by 4 2.27 × 10^3 N
Allow their central force **B1**

(iii) $mgh = ½\, mv^2$ **B1**

Condone: Use of $v = 2\pi fA$ (max2)

9.8×44 = 0.5 v^2 Allow 45 in substitution **B1**

Condone 22 m s^{-1}

29.4 m s^{-1} (Use of 45 gives 29.7)

B1

106 km h^{-1} (their m s^{-1} correctly converted)
Or compares with 33 m s^{-1}

B1

(iv) 1/16th(0.625) % of KE left if correct

M1

Allow 1/8 (0.125)or 1/32(0.313)

KE at start = 5.6×10^4 J or states energy ∝ speed^2 so speed is ¼

M1

Allow for correct sub^n E =½ 280 × 20^2 x factor from incorrect number of swings calculated correctly

Final speed calculated = 5 m s^{-1}

A1

Must be from correct working

[17]

7 **D**

[1]

8 **A**

[1]

9 B

[1]

10 C

[1]

Examiner reports

1 Question (a) was an extended calculation and 60% of students were able to calculate the mass or spring constant. The ‘consequential error’ principle was applied and so an incorrect value could be used to calculate the other value for full credit.

The next part also resulted in many good answers and although students did not need to use their values, full credit was given for correct calculations using calculated mass and spring constant, even if one or both of these were incorrect.

Question (c) required students to identify two differences that would occur when the mass attached to spring B oscillated in oil. Identifying the differences proved to be straightforward but giving the reason for this far less so. The most common difference given was the decrease in amplitude, with the next most popular being the shift in the peak amplitude to a higher frequency.

3 Part (a)(i) offered an easy mark for naming the two types of potential energy involved in an oscillating suspended mass-spring system. “Gravitational potential energy” is clear and unambiguous, but a variety of terms appeared to be in use for the energy stored by a stretched spring. “Elastic potential energy” was the expected term, but “strain energy” was equally acceptable. For obvious reasons “stored energy” (when unqualified) was not.

Those who had concentrated on the wording of the question in part (a)(ii) – especially on “energy changes”, “one complete oscillation” and “starting at its lowest point” – were able to give good concise answers. Far too many of the students attempted to consider the absolute values of elastic and gravitational energies during an oscillation, which usually led them into confusion and irrelevance. Many answers stated that elastic potential energy would increase as the mass moved above the equilibrium position because the spring would be compressed. Inevitably a lot of answers described only half of an oscillation: if the energy types and changes were correctly described even this was given 1 mark. Answers which did not refer to the kinetic energy of the system were not credited.

In part (b)(i) those who appreciated that the total potential energy of the system passes through two maxima per oscillation, one at each amplitude, came up with the expected 0.8 s. Because they did not understand this, getting on for half of the students gave 0.4 s. There were consequences in the later parts of this question, where incorrect values from part (b)(i) were generally accepted as a basis for the work that followed. A small minority of the graphs drawn in part (b)(ii) were triangular, but the majority represented some form of sinusoidal variation. Whether this agreed with the expected period (0.8 s), and was a negative cosine curve, proved to be more testing issues.

The time period calculation in part (c)(i) was straightforward. This was rewarding for those who could substitute mass and period values correctly and then calculate the expected value. N m^{-1} was the only answer accepted for the unit of k. The amplitude calculation in the final part of the question was often done well. Students who had made an error over the time period earlier were unable to show that the value of the amplitude would be about 40 mm, so were limited to a mark of 3 out of 4. Working from T = 0.4 s, many of these answers arrived at a value of 21.5 mm before the students introduced a mystery factor of 2 to end up with “about 40 mm”!

4 Except for part (c), this question was on material familiar to most candidates, so the marks awarded were generally high. In part (a) definitions of simple harmonic motion, in terms of the two features of the acceleration of a body moving with shm, were generally well known. Part (b)(i) presented a greater challenge for candidates who were unsure about how to handle significant figures. All the data required to complete the calculation in this question was available to 3SF. Therefore all the working should have been to at least 3SF and the answer should be quoted to 3SF. Candidates who used g = 9.8 instead of g = 9.81 lost a mark, and a further mark was lost if the final answer was not expressed to 3SF. Some of the weaker candidates did not appear to know the difference between period and frequency. Part (b)(ii) required a straightforward application of $a = (–)\ (2\pi f)^2 \times$ and presented few problems.

The solution of the question in part (c), involving the minimum time period between 'in phase' positions of two pendulums of different frequencies, could not be arrived at by a standard method that most candidates would have encountered. Consequently the explanations of how the answer had been arrived at were often unsatisfactory. Trial and error seemed to be a popular approach, sometimes leading to the correct answer of 38s without any working at all.

Probably the most satisfactory solution is to recognise that the *shorter* pendulum must make one more oscillation than the longer pendulum in the required time, hence the number of oscillations of the *longer* pendulum is given by (n + 1) × 1.9 = n × 2.0. Another successful approach follows from appreciating that Δt = 0.1s, so the number of oscillations required of the shorter pendulum is 2.0 / 0.1 = 20 (alternatively 1.9 / 0.1 gives 19 oscillations of the longer pendulum). This approach sometimes led to an incorrect conclusion, such as 19 × 1.9 = 36.1s, or 20 × 2.0 = 40s.

5 (a) (i) Although only 3 of 5 possible statements were required for full marks, many students found this question difficult. Some believed that the tension at the lowest point was mg, totally ignoring centripetal force. Some stated the weight acts at different angles as the trapeze swings, and some though the centripetal force was constant.

(ii) A few attempts to change the subject of the formula were incorrect, but most students were awarded full marks.

(b) (i) Correct answers were reached by those who used the formula $V = 2\pi fA$, but those that chose the energy method invariably used the wrong value of height (0.48m)

(ii) Most students had difficulty here. Some used the incorrect formula $x = A \cos(2\pi ft)$ and others used Pythagoras in an attempt to find the horizontal distance. Of those who chose an equation of motion, some chose inappropriately and were unable to find, the time (1.11 s).

(c) (i) Many students had little idea of how to tackle this question. The most straight forward correct method was to calculate the ratio of successive amplitudes. Those who attempted to find the half-life were expected to sketch the curve on the graph on (figure 2). The most difficult and rarely correctly executed method was to calculate k in = $A_o e^{-kt}$.

(ii) Most students completely missed the point that the simple pendulum is effectively shortened and so the period is reduced. Those using $T = 2\pi \sqrt{m/k}$ and others claiming that air resistance affected T gained no marks. There were lots of non-committal answers failing to state whether T increased or decreased.

6 (a) (i) A minority of the candidates realised that the PE reaches a maximum twice per cycle so identifying a period of 0.4 s was most common.

(ii) Most drew diagrams that showed the KE at peak when the PE was at zero. The majority however either did not know or were careless in showing that PE = KE when the graphs cross over.

(b) (i) This was a straightforward question which produced many correct answers. Poor algebra was a problem for those who were unsuccessful.

(ii) Most were able to calculate the speed in m su^{-1} and go on to find the centripetal force. Fewer realised that the weight of the riders and harness had to be added to this to find the sum of the tensions in the cables. Many however did remember to divide what they thought was the tension by 4.

(iii) Determining the number of half oscillations proved difficult for some candidates but most appreciated the need to do a $(0.5)^n$ type calculation to find the final energy. A common error was to multiply the fractional change (1 / 16) by the initial speed of 20 m s^{-1}.

7 48.0% correct

8 66.6% correct