Mark schemes

1 (a) Total mass of spacecraft = 3050 kg

Change in PE = $\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 3050}{6400 \times 10^{3}}$

1.9×10^{11}(J)

2 sf

condone errors in powers of 10 and incorrect mass for payload

Allow if some sensible working

4

(b) Chemical combustion of propellant / fuel or gases produced at high pressure

Gas is expelled / expands through nozzle

Change in momentum of gases escaping

equal and opposite change in momentum of the spacecraft

Thrust = rate of change of change in momentum

Max 3

N3 in terms of forces worth 1

3

(c) 0.031(4) (m s^{-2})

1

(d) Use of rocket equation

$v = 1200 \ln \frac{3050}{1330}$

996 (m s^{-1})

Condone 1000 (m s^{-1})

3

(e) (i) Use of correct mass 108 kg

$$F = \frac{6.67 \times 10^{-11} \times 1.1 \times 10^{13} \times 108}{(2 \times 10^{3})^2}$$

0.0198 N

Allow incorrect powers of 10 and mass

3

(ii) Use of $v = \sqrt{\frac{2GM}{r}}$

Correct substitution $v = \frac{2 \times 6.67 \times 10^{-11} \times 1.1 \times 10^{18}}{2 \times 10^{8}}$

0.86 (m s^{-1})

Recognisable mass – condone incorrect power of 10

3

(iii) Impulse = 25 N × 4.8 = 120 N s

(120 = 108 *v* so) Velocity = 1.1 m s^{-1}

Clear conclusion

ie explanation/comparison of calculated velocity with escape velocity from **(e)(ii)**

May use F = ma approach

3

[20]

2 (a) (i) (Minimum) Speed (given at the Earth's surface) that will allow an object to leave / escape the (Earth's) gravitational field (with no further energy input)

Not gravity

Condone gravitational pull / attraction

B1

1

(ii) $\frac{1}{2} mv^2 = \frac{GMm}{r}$

B1

Evidence of correct manipulation

At least one other step before answer

B1

2

(iii) Substitutes data and obtains $M = 7.33 \times 10^{22}$(kg)

or

Volume = $(1.33 \times 3.14 \times (1.74 \times 10^{6})^3$ or 2.2×10^{19}

or $\rho = \frac{3v^2}{8\pi G r^2}$

C1

3300 (kg m^{-3})

A1

2

(b) (Not given all their KE at Earth's surface) energy continually added in flight / continuous thrust provided / can use fuel (continuously)

B1

Less energy needed to achieve orbit than to escape from Earth's gravitational field / it is not leaving the gravitational field

B1

2

[7]

3 (a) Idea that both astronaut and vehicle are travelling at same (orbital) speed or have the same (centripetal) acceleration / are in freefall

Not falling at the same speed

B1

No (normal) reaction (between astronaut and vehicle)

B1

2

(b) (i) Equates centripetal force with gravitational force using appropriate formulae

E.g. $\frac{GMm}{r^2} = \frac{mv^2}{r}$ or $mr\omega^2$

B1

Correct substitution seen e.g. $v^2 = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{\text{any value of radius}}$

B1

(Radius of) 7.28×10^6 seen or $6.38 \times 10^6 + 0.9 \times 10^6$

B1

7396 (m s^{-1}) to at least 4 sf
Or $v^2 = 5.47 \times 10^7$ seen

B1

4

(ii) $\Delta PE = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.68 \times 10^4 (1 / (7.28 \times 10^6) - 1 / (6.78 \times 10^6))$

C1

-6.8×10^{10} J

C1

$\Delta KE = 0.5 \times 1.68 \times 10^4 \times (7700^2 - 7400^2) = 3.81 \times 10^{10}$ J

C1

$\Delta KE - \Delta PE = (-)\ 2.99 \times 10^{10}$ (J)

A1

OR

Total energy in original orbit shown to be $(-)GMm / 2r$ or $mv^2 / 2 - GMm / r$

C1

Initial energy
$= -6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.68 \times 10^4 / (2 \times 7.28 \times 10^6)$
$= 4.59 \times 10^{11}$

C1

Final energy
$= -6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.68 \times 10^4 / (2 \times 6.78 \times 10^6)$
$= 4.93 \times 10^{11}$

3.4×10^{10}(J)

Condone power of 10 error for C marks

A1

4

[10]

4 (a) (i) force per unit mass ✓
a vector quantity ✓

Accept force on 1 kg (or a unit mass).

2

(ii) force on body of mass *m* is given by $F = \frac{GMm}{(R+h)^2}$ ✓

gravitational field strength $g\left(=\frac{F}{m}\right) = \frac{GM}{(R+h)^2}$ ✓

For both marks to be awarded, correct symbols must be used for M and m.

2

(b) (i) $F\left(=\frac{GMm}{(R+h)^2}\right) = \frac{6.67\times10^{-11}\times5.98\times10^{24}\times2520}{((6.37\times10^{6})+(1.39\times10^{7}))^2}$ ✓

= 2.45×10^3 (N) ✓ to **3SF** ✓

1st mark: all substituted numbers must be to at least 3SF.

If 1.39×10^7 is used as the complete denominator, treat as AE with ECF available.

3rd mark: ***SF mark is independent.***

3

(ii) $F = m\omega^2 (R + h)$ gives $\omega^2 = \dfrac{2450}{2520 \times 2.03 \times 10^7}$ ✓

from which $\omega = 2.19 \times 10^{-4}$ (rad s^{-1}) ✓

time period $T\left(=\dfrac{2\pi}{\omega}\right) = \dfrac{2\pi}{2.19 \times 10^{-4}}$ **or** = 2.87 ✓ 10^4 s ✓

[**or** $F = \dfrac{mv^2}{R+h}$ gives $v^2 = \dfrac{2.45 \times 10^3 \times \left(\left(6.37 \times 10^6\right) + \left(13.9 \times 10^6\right)\right)}{2520}$ ✓

from which v = 4.40 ✓ 10^3 (m s–1) ✓

time period $T\left(=\dfrac{2\pi\left(R+h\right)\right)}{v}\right) = \dfrac{2\pi \times 2.03 \times 10^7}{4.40 \times 10^3}$ **or** = 2.87×10^4 s ✓]

[**or** $T^2 = \dfrac{4\pi^2 (R+h)^3}{GM}$ ✓

$= \dfrac{4\pi^2 ((6.37 \times 10^6) + (13.9 \times 10^6))^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}$ ✓

gives time period T = 2.87×10^4s ✓]

$= \dfrac{2.87 \times 10^4}{3600}$ = 7.97 (hours) ✓

number of transits in 1 day = $\dfrac{24}{7.97}$ = 3.01 (≈ 3) ✓

Allow ECF from wrong F value in (i) but mark to max 4 (because final answer won't agree with value to be shown).

First 3 marks are for determining time period (or frequency). Last 2 marks are for relating this to the number of transits.

Determination of f = 3.46×10^{-5} (s^{-1}) is equivalent to finding T by any of the methods.

5

(c) acceptable use ✓

satisfactory explanation ✓

e.g. monitoring weather **or** surveillance:

whole Earth may be scanned **or** Earth rotates under orbit

or information can be updated regularly

or communications: limited by intermittent contact

or gps: several satellites needed to fix position on Earth

Any reference to equatorial satellite should be awarded 0 marks.

2

[14]

5 (a) (i) Use of $F – GMm/r^2$

C1

Allow 1 for
-correct formula quoted but forgetting square in substitution

Correct substitution of data

M1

-missing m in substitution

491 (490)N

A1

-substutution with incorrect powers of 10

Condone 492 N,

(ii) Up and down vectors shown (arrows at end) with labels

B1

allow W, mg (not gravity); R
allow if slightly out of line / two vectors shown at feet

up and down arrows of equal lengths

B1

condone if colinear but not shown acting on body

In relation to surface W ≤ R (by eye) to allow for weight vector starting in middle of the body

Must be colinear unless two arrows shown in which case R vectors ½ W vector(by eye)

(b) (i) Speed = $2\pi r / T$

B1

Max 2 if not easy to follow

2π6370000 / (24 × 60 × 60)

B1

463 m s^{-1}

B1

Must be 3sf or more

(ii) Use of $F = mv^2/r$

C1

Allow 1 for use of $F = mr\omega^2$ with ω= 460

1.7 (1.66 – 1.68) N

A1

(iii) Correct direction shown
(Perpendicular to and toward the axis of rotation)
NB – not towards the centre of the earth

B1

(c) Force on scales decreases / apparent weight decreases
Appreciates scale reading = reaction force

C1

The reading would become 489 (489.3)N or reduced by 1.7 N)

A1

Some of the gravitational force provides the necessary centripetal force

B1

or $R = mg - mv^2/r$

[14]

6 (a) zero potential at infinity (a long way away)

B1

energy input needed to move to infinity (from the point)
work done by the field moving object from infinity
potential energy falls as object moves from infinity

B1

2

(b) Any pair of coordinates read correctly

C1

±1/2 square

Use of E_p or $V = (-)\dfrac{GM}{r}$

C1

Rearrange for M

6.4 (±0.5) × 10^{23} kg

A1

3

(c) Reads correct potential at surface of Mars = −12.6 (MJ)

C1

or reads radius of mars correctly (3.5×10^6)

equates to ½ v^2 (condone power of 10 in MJ)

C1

use of $v = \sqrt{(2GM/r)}$ *with wrong radius*

5000 ± *20* m s^{-1} (condone 1sf e.g. 5 km s^{-1})

A1

e.c.f. value of M from (b) may be outside range for other method 6.2 $\times 10^{-9} x \sqrt{\text{their } M}$

3

(d) Attempts 1 calculation of *Vr*

B1

Many values give 4.2.... so allow mark is for reading and using correct coordinates but allow minor differences in readings
Ignore powers of 10 but consistent

Two correct calculation of *Vr*

B1

Three correct calculations with conclusion

B1

3

[11]

7 **D**

[1]

8 **C**

[1]

9 **A**

[1]

10 **C**

[1]

Examiner reports

1

(a) Most students gave the answer to 3 significant figures, although 2 sf was what was required. The correct mass (3050 kg) was chosen by those who used the correct formula, but some students used no mass in calculating the potential only.

(b) Many stated that the propellant/fuel was ejected through the nozzle. The statements about the momentum of the exhaust gases were often confused. The most popular way of deriving thrust was by attempting to use Newton's 3 rd law but the statements were often incomplete.

(c) The simple use of F = ma was easily achieved by most students.

(d) Although some students attempted to use conservation of momentum, most realised that the rocket equations was needed. There is the same confusion over the meaning of the symbols v_f and m_f. Some used m_f = 1720 kg instead of 1330 kg, and others, after correctly calculating v_f = 996 m s^{-1}, went on to subtract this from the exhaust gas speed, thus sacrificing a mark.

(e) (i) Most students chose the correct formula, but many forgot to square the radius, and others chose the wrong mass. The original mass of the spacecraft (3050 kg) was the most popular erroneous value, although even the mass of the Earth was seen occasionally.

(ii) Nearly everyone started with the correct formula but two common errors ensued. Some forgot to take the square root and others did not convert 2.0 km to meters. Also some gave the answer to 1 sf (0.9 m s^{-1}) thus losing a mark.

(iii) Those who calculated that the velocity change of the probe was 1.1 m s^{-1} followed with the right conclusion. Some students used the wrong mass but could still gain the third mark with a correct comparison.

2

(a) (i) Well done.

(ii) Candidates scored 2 or zero. The latter invariably used centripetal force = gravitational force.

(iii) Many promising calculation were ruined by failure to cube the radius when finding the volume.

(b) Most candidates did not realise that escape velocity was not needed because the rocket was not escaping!

3

(a) Most candidates mentioned the lack of reaction force but some answers were spoilt by claiming that there was no resultant force or even no gravity.

(b) (i) Each step needed to be clearly shown, starting with the statement that gravitational force = centripetal force.

There were several cases of the use of 900 km for the radius.

(ii) Common errors were treating potential energy as positive, the use of the wrong radii and the use of mgh.

4 The definition in part (a)(i) was well known. Because the quantity concerned is called gravitational field *strength*, there was frequent confusion as to whether it is a vector or a scalar, with many answers being crossed out and changed. Part (a)(ii) was also generally very rewarding. The main problem was a failure to show how the terms from the data booklet equations (m_1, m^2 and r) translated into the terms in the question (m, M, R and h). In the derivation, some students cancelled M instead of m. However, others had so little confidence in their use of algebra that they could make little progress even in a simple derivation such as this.

Part (b)(i) caused few problems and marks were generally high. Sometimes incorrect values had been extracted from the data booklet for the mass and radius of the Earth. Three significant figures were expected in the answer; therefore a minimum of three significant figures should also have been used in the substitution and working. When $h = 1.39 \times 10^7$ was used as the radius of the orbit one mark was lost and the value of the force thus obtained was carried forward to make most marks available in part (b)(ii). Part (b)(ii) offered a very wide range of approaches to enable students to show that the satellite would make three transits of Earth in every 24 hours. Apart from the three alternatives given in the mark scheme (all of which were frequently seen) a very concise calculation showed that a satellite with an angular speed of 2.19×10^{-4} rad s^{-1} would move through an angle of 18.9 rad in one day, equating to ($18.9 / 2\pi =$) 3.01 transits.

Use of polar orbiting satellites for monitoring the Earth (weather forecasting, spying, surveying, etc.) were well known in part (c), although some students confused the application with an equatorial geosynchronous satellite. Explanations of the application were often less satisfactory: reference to the rotation of Earth beneath the orbit, allowing the whole surface to be scanned, was the key here. The ability to provide regular updates of the information obtained was also an acceptable explanation. Students who mentioned the use of the polar satellite for communications gained the first mark but were usually unable to point out its limitations, caused by intermittent contact.

5 (a) (i) Most candidates were able to make good progress with this calculation and there were many correct answers.

(ii) Many attempts were unconvincing and frequently carelessly drawn. Weight and reaction forces were often shown as not being collinear. Some showed a reaction force at one of the feet but not the other. That the length of a vector should represent magnitude was not realised by many candidates.

(b) (i) A good proportion of correct approaches were seen but many candidates seemed unsure what equation to use so quoted some that were not relevant. Good structure in a mathematical argument is an important skill in all problems but even more so in 'show that' type questions were marks are awarded for each step.

(ii) Again there was a good proportion of correct response. Some candidates used $F = mr\omega^2$; and 460 m s^{-1} for ω.

(iii) Misunderstanding about centripetal force was common here and there were relatively few correct answers. The majority showed the force acting toward the centre of the Earth. Whilst a component of this force <u>provides</u> the centripetal force, the direction of the centripetal force is toward the centre of rotation which in the diagram is perpendicular and toward the axis of rotation of the Earth.

(c) There were very good answers from candidates who understood that the scales read the reaction force.There were many who knew the formula $mg - R = mv^2 / r$ but thought that the scales would record *mg* and assumed *R* to remain constant so that the centripetal force increased the scale reading.

6 (a) Most candidates knew the infinity reference point but many had difficulty explaining the negative sign. A statement such as 'work has to be done to move a mass from infinity' begs the question whether work being done within the Earth-mass system energy by the gravitational field to reduce the potential energy or whether the work is being done by an outside energy source.

(b) A high proportion of candidates completed this successfully either using the potential formula or by considering potential energy changes. Use of the force formula was a common error.

(c) Most read the radius from the graph and used the value of mass from (b) rather than the method of reading the potential energy (−)12.6 MJ from the graph and equating it to ½v^2. Using arbitrary energies or incorrect reading from the graph were common errors.

(d) The majority appreciated what was an acceptable test and almost half the candidates scored full marks. Others usually used only two points on the curve or used three appropriately but left it to the examiner to make the conclusion from the values obtained.

7 36.6% correct

8 30.8% correct

9 58.5% correct