Mark schemes

1 (a) (mark should be at the equilibrium position) since this is where the mass moves with <u>greatest speed</u> [transit time is least] ✓

1

(b) (i) mean time for 20T (from sum of times ÷5) = 22.7 (s)$_1$ ✓ (minimum 3sf)

uncertainty (from half of the range) = 0.3 (s) $_2$ ✓ (accept trailing zeros here)

percentage uncertainty

(from $\frac{0.3}{22.7} \times 100$) [$\frac{100}{5} \times \sum \frac{0.3}{20T}$] = 1.3 (22)%$_3$ ✓

(allow full credit for conversion from 20T to T, e.g. 1.135 = $_1$✓ 0.015 =$_2$ ✓ ecf for incorrect $_1$ ✓ and / or $_2$ ✓ earns $_3$ ✓

3

(ii) natural frequency (from $\frac{20}{22.7}$ and minimum 2 sf) = 0.88 (1)Hz [accept s^{-1}] ✓

(ecf for wrong mean 20T; accept ≥ 4 sf)

1

(c) (i) linear scale with at least 3 evenly-spaced convenient values (i.e. not difficult multiples) marked; the intervals between 1 Hz marks must be 40 ± 2 mm (100 ± 5 mm corresponds to 2.5 Hz) ✓

(ecf for wrong natural frequency: 100 ± 5 mm corresponds to $\frac{2.5f}{0.88}$ Hz)

1

(ii) 4 mm [allow ± 0.2 mm] ✓

1

(d) (i) student decreased intervals [smaller gaps] between [increase frequency / density of] readings (around peak / where A is maximum) ✓ ✓

[student took more / many / multiple readings (around peak) ✓]
(reject bland ‘repeated readings’ idea; ignore ideas about using data loggers with high sample rates)

2

(ii) new curve starting within ± 1 mm of A = 4 mm, f = 0 Hz with peak to right of that in Figure 3
(expect maximum amplitude shown to be less than for 2 spring system but don't penalise if this is not the case; likewise, the degree of damping need not be the same (can be sharper or less pronounced)
Peak at $\sqrt{2}$ value given in **(b)(ii)**; expect 1.25 Hz so peak should be directly over 50 ± 5 mm but take account of wrongly-marked scale ✓

2

[11]

2 (a) *forced vibrations:*

repeated upwards and downwards movement ✓
vibrations at frequency of support rod ✓
amplitude is small at high frequency **or** large at low frequency ✓
correct reference to phase difference between displacements of driving and forced vibrations ✓

Acceptable references to phase differences:

*Forced vibrations − when frequency of driver » frequency of driven, displacements are out of phase by (almost) π radians or 180° (**or** ½ a period) **or** when frequency of driver « frequency of driven, displacements are (almost) in phase. [Accept either].*

[Condone >, < for », «].

resonance:

frequency of support rod **or** driver is equal to natural frequency of (mass-spring) system ✓
large (or maximum) amplitude vibrations of mass ✓
maximum energy transfer (rate) (from support rod to mass-spring system) ✓
correct reference to phase difference between displacements of driving and driven vibrations at resonance ✓

*Resonance − displacement of driver leads on displacement of driven by π / 2 radians or 90° **or** ¼ of a period (or driven lags on driver by π / 2 radians or 90° **or** ¼ of a period).*

[Condone phase difference is π / 2 radians or 90°].

max 4

(b) (i) cone oscillates without ring *(ticked)*

Only one box to be ticked.

1

(ii) damping is caused by air resistance ✓
area is the same whether loaded or not loaded ✓
loaded cone has more kinetic energy **or** potential energy **or** momentum (at same amplitude) ✓
smaller <u>proportion</u> (or <u>fraction</u>) of (condone less) energy removed per oscillation from loaded cone (or vice versa) ✓
inertia of loaded cone is greater ✓

Award marks for correct physics even when answer to (b)(i) is incorrect.

max 3

[8]

3 (a) forced vibrations or resonance **(1)**

1

(b) reference to natural frequency (or frequencies) of structure **(1)**
driving force is at same frequency as natural frequency of structure **(1)**
resonance **(1)**
large <u>amplitude</u> vibrations produced or large energy transfer to structure**(1)**
could cause damage to structure [or bridge to fail] **(1)**

max 4

(c) stiffen the structure (by reinforcement) **(1)**
install dampers or shock absorbers **(1)**
[or other acceptable measure e.g. redesign to change natural frequency or increase mass of bridge or restrict number of pedestrians]

2

[7]

4 (a) vibrations are forced when periodic force is applied **(1)**

frequency determined by frequency of driving force **(1)**

resonance when frequency of applied force = natural frequency **(1)**

when vibrations of large amplitude produced

[or maximum energy transferred at resonance] **(1)**

(max 3)

(b) (i) damping when force opposes motion [or damping removes energy] **(1)**

(ii) damping reduces sharpness of resonance

[or reduces amplitude at resonant frequency] **(1)**

(2)

[5]

5 (a) (i) *free:* system displaced and left to oscillate **(1)**

(ii) *forced:* oscillation due to (external) periodic driving force [or oscillation at the frequency of another vibrating system] **(1)**

2

(b) (i) $k = \frac{3000}{5.0 \times 10^{-2}} = 6.0 \times 10^{4}$ Nm^{-1} **(1)**

(ii) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{9000}{g \times 6.0 \times 10^{4}}}$
giving 0.78 s **(1)**

3

(c) (i) $t = \frac{s}{v} = \frac{16}{20} = 0.80$ s **(1)**

(ii) time ≅ period of free oscillations, resonance **(1)**
i.e. large amplitude oscillations **(1)**

3

[8]

6 (a) (i) $r = 0.012$ (m) **(1)**
(use of $v = 2\pi fr$ gives) $v = 2\pi 50 \times 0.012$ **(1)**
= 3.8 m s^{-1} **(1)** (3.77 m s^{-1})

(ii) correct use of $a = \frac{v^2}{r}$ or $a = \frac{3.8^2}{0.012}$ **(1)**

= 1.2 × 10^{3} m s^{-2} **(1)**

[or correct use of $a = \omega^2 r$]
(allow C.E. for value of v from (i)

5

(b) panel resonates **(1)**
(because) motor frequency = natural frequency of panel **(1)**

2
QWC 2

[7]

7 (a) accelerating to left **(1)**
net force to left on M due to compression on right and tension on left **(1)**/p>

(2)

(b) slider is at 33/50 of length **(1)**
(uniform track so) resistance ∝ length **(1)**

$V = \frac{33}{50} \times 5.0$ (V) **(1)** = 3.3 V **(1)**

ΔV = +0.8V **(1)**

(max 4)

(c) damping **(1)**
to prevent (reduce) oscillation of mass (during changes in motion) **(1)**

(2)

[8]

8 C

[1]

9 D

[1]

10 **B**

[1]

11 A

[1]

12 B

[1]

13 A

[1]

14 C

[1]

15 A

[1]

16 B

[1]

17 D

[2]

18 C

[1]

Examiner reports

1 Encouraging numbers knew to say in **(a)** that the fiducial mark is placed at the equilibrium position because this is where 'the transit time is least' or 'the speed of the mass is greatest' but it was common to find statements such as 'this is where the mass must pass through in every oscillation' which attracted no credit.

Part **(b)(i)** and **(b)(ii)** were routine calculations in which candidates were usually successful although errors for the frequency arose due to rounding down earlier in the calculation. Full credit could be earned if the correct result for percentage uncertainty was reached using T rather than $20T$ but we withheld a mark in **(b)(ii)** if no unit was seen.

Part **(c)(i)** was very good and we took account of an incorrect **(b)(ii)** answer that led to an unexpected scale. We expected the scale to be linear and at least three convenient intervals marked. For the expected of 0.88 Hz result in **(b)(ii)** the 1 Hz intervals on the axis should have been 40 mm apart.

Many stated in **(c)(ii)** that the initial amplitude of X was 4 mm but a common error was to read off the amplitude at resonance (95 mm).

Part **(d)** was the most discriminating part of this question. In **(d)(i)** many grasped the idea behind the question but simply saying 'take more readings' was insufficient and only gained one mark. Some candidates avoided any ambiguity by saying 'the intervals between the frequency readings around the peak were reduced' to earn full credit. A common but unsuccessful idea was to assume a data logger was in use and to say that the sample rate was increased around the peak.

In **(d)(ii)** saw that removing one spring doubled the stiffness and increased the resonant frequency by a factor of $\sqrt{2}$. The new curve now had a resonant peak at about 1.25 Hz which for an axis with the expected calibration was 50 mm along the axis. Some spoiled their answer by starting the curve at 0 Hz with amplitude less than 4 mm.

Unlike last year the full mark range was utilised and the question discriminated very well. Those at the A/B boundary usually earned between 7 and 9 out of 11 and E/U candidates often scoring between 4 and 6.

2 In part (a) most students knew far more about resonance than about forced vibrations. Relatively few were able to point out that the forced vibrations of the mass-spring system would have the same frequency as the oscillations of the support rod, and knowledge of the relative amplitudes and phases of the vibrations were rarely approached successfully. These features of a mass-spring system may be readily demonstrated by oscillating the system vertically at low and high frequencies by hand.

The main features of the resonating system were well known, but ambiguous expressions often limited the mark that could be awarded. An example of this was “the system oscillates at its natural frequency”. Without any reference to the driving frequency, this could mean free vibrations rather than resonant vibrations. The mark scheme adopted for part (a) meant that even those students who knew nothing about forced vibrations could score highly by their knowledge of resonance.

Part (b)(i), which was partly synoptic, proved to be a much sterner test than had been expected. Fewer than half of the students ticked the correct box, failing to realise that the cone without the ring would be the most heavily damped. The explanations presented in (b)(ii) revealed widespread misunderstanding. Many students were more concerned with the effect of the ring on the period of oscillation rather than on the decay of successive amplitudes. Resorting to the simple pendulum equation, where time period is independent of mass, led many to the conclusion that the mass of the ring would have no effect. Observation of simple experimental phenomena, such as looking at objects with different surface area-to-mass ratios falling through air, would readily demonstrate the effect of air resistance and the factors which affect the removal of energy from a moving body. Hardly any of the answers approached the most important aspects: that the loaded cone gains more kinetic energy after being given the same displacement, and that a *smaller proportion* of the energy of the oscillating system would be removed *per oscillation* from the loaded cone.

3 Candidates’ knowledge of forced vibrations, resonance and damping was rather better than might have been anticipated when this question was set. Consequently most candidates achieved high marks.

The Millennium footbridge has turned out to be a most complex structure, somewhat removed from the temporary “advancing army” type of footbridge traditionally considered in text books. When it was first opened, walkers on the bridge subconsciously adjusted their steps so that they were synchronised with lateral vibrations of the bridge span. In effect this was a feedback phenomenon in which the bridge vibrations were being passed back to the driving forces when there were a large number of people relative to the mass of the bridge and where the level of damping was low. Candidates could not be expected to be familiar with the full detail of this, and consequently the examiners took the view that “resonance” was an acceptable alternative to “forced vibrations” as an answer to part (a).

Part (b) was usually very rewarding. Clear reference to the frequency of the driving force was not always made, however. The examiners were also looking for mention of the amplitude of vibrations, rather than just the size of them. Many good, and some ingenious, answers were seen in part (c). The need for engineers to make the changes was not always appreciated. For example, some candidates suggested that the pedestrians should walk out of step. Neither were the limitations on the materials available to build a bridge readily recognised by some candidates; changing the natural frequency of the material of the bridge would not be a very practical solution. There were also candidates who thought that shortening the bridge would be an acceptable strategy! Despite these unsatisfactory responses, the majority of the candidates knew enough to suggest measures such as improved damping and a stiffer structure.

4 Answers to this question showed that even average candidates experienced difficulty in explaining themselves, and the weaker candidates struggled considerably. The knowledge that forced vibrations are caused by a driving force which is *periodic* was often omitted. Most candidates had a fair idea of resonance.

In part (b)(i) only a minority of candidates could explain that damping removes *energy* or occurs when the motion is opposed by a *force*. Very few candidates had the knowledge to score in part (b)(ii).

5 The better candidates answered this question well, particularly parts (b) and (c).

There were a few good answers to part (a), but there were also many poor ones. Candidates often wrote too much. Attempts to say that no forces acted in a free vibration often implied that no initial force was required or that no damping was allowed. Many candidates thought that free vibrations occurred at the natural frequency whilst forced ones did not, and there was much confusion with resonance. Those candidates who simply gave a good example in each case were given some credit.

Full marks were often scored in parts (b) and (c). Those candidates who knew how to calculate k in part (b)(i) sometimes used 9000 N as the load or omitted the unit. An appreciable number of candidates forgot $F = ke$ and tried to use $T = 2\pi\sqrt{m/k}$ to calculate k, assuming T from part (b)(ii).

Many candidates drew the correct conclusion in part (c) that, because the calculated time was close to the period of the spring, there would be resonance and large amplitude oscillations. Other candidates thought that the small difference in time (0.02s) was important and, if their ideas were correct, they were given some credit.

6 Many candidates scored all three marks in part (a)(i), but some were careless and used the given value of diameter for the radius or did not include π in their calculations. A few candidates lost the final mark as a result of giving the answer to too many significant figures.

In part (ii), although some candidates confused speed with angular velocity, many correct answers were seen using $\frac{v^2}{r}$ or $\omega^2 r$. Candidates who repeated the error of using the value of the diameter rather than the radius were not penalised again.

In part (b) most candidates knew that the effect was due to resonance but not all of them were able to provide a clear explanation of why resonance occurred at a particular rotational speed of the motor.

10 This question, on coupled pendulums, had appeared in the 2009 examination. The results on that occasion were surprisingly disappointing, with fewer than 40% of correct responses. In 2016 more of the students selected the correct resonant pendulum (B), but it was only 53% of them. One third of the students chose pendulum A, the longest one.

11 The effect of heavier damping on the amplitude of vibration of an oscillating object when close to resonance was the subject tested by this question. 74% of the students appreciated correctly that the amplitude would be reduced at all frequencies, but 18% considered that heavier damping would be effective only at frequencies above resonance (distractor C).

12 This question presented students with four amplitude–frequency graphs for a resonant system, from which they were to select the best illustration of light damping. Around half of the students had this correct; incorrect answers were evenly distributed around the distractors.

13 This question required students to choose an incorrect statement about a mechanical system oscillating at resonance. The question had been used in a 2004 examination when it was found to be easy. It proved to be slightly easier this time, with 72% of responses correct. None of the three distractors attracted a response that was significantly higher than the others.

14 Candidates for this test were slightly less familiar with the direction of the damping force acting on a vibrating system, required in this question, than those in 2005 when this question was last used. On both occasions about three-fifths of candidates made the right choice. Incorrect responses were fairly evenly spread across the other distractors.

15 This question, on forced vibrations, had a facility of 59% and did not discriminate very well. Distractor B, where a phase relation was involved, attracted 23% of the candidates. This again may be an indication of a misunderstanding of phase angles, because the angle in the situation described is 90°, not 180°.

16 This question was concerned with the damping force in a vibrating system. The 2005 candidates evidently had a better understanding of this topic than those who answered this question on an Advanced paper five years earlier, because the facility advanced from 57% then to 62% this time. Although there was no particularly strong distractor, the question did not discriminate very well.

17 This question was concerned with resonance and damping. 69% of the candidates arrived at the correct response, a slight improvement on the pre-test result. The question had the lowest discrimination index (0.36) of any of the questions on this paper, possibly because it was one of three questions in this test that required candidates to identify an *incorrect* statement.